1. Potenzgesetze – vereinfache – ohne Taschenrechner!

a) $\sqrt[3]{12}\cdot\sqrt[5]{12} = 12^{\frac{1}{3}}\cdot 12^{\frac{1}{5}} = 12^{\left(\frac{1}{3}+\frac{1}{5}\right)} = 12^{\frac{8}{15}}$

b) $\frac{\sqrt[3]{30}}{\sqrt[3]{\frac{15}{4}}} = \sqrt[3]{\frac{30\cdot 4}{15}} = \sqrt[3]{8} = 2$

c) $\sqrt[9]{\sqrt[3]{512}} = \sqrt[9]{8} = \sqrt[3]{2}$

2. Fasse zu einer Potenz zusammen!

a) $\frac{\left(\sqrt{5}\right)^{,3}}{\left(\sqrt{5}\right)^{\frac{7}{10}}} = \frac{\left(5^{\frac{1}{2}}\right)^{\frac{13}{10}}}{\left(5^{\frac{1}{2}}\right)^{-\frac{7}{10}}} = \frac{5^{0,65}}{5^{-0,35}} = 5$

b) $\left(x^{\frac{5}{2}}+2\right)^2 = x^5 + 4x^{\frac{5}{2}} + 4$

c) $\sqrt[3]{x^2}\cdot\sqrt[6]{x^4} = x^{\frac{2}{3}}\cdot x^{\frac{4}{6}} = x^{\frac{2}{3}}\cdot x^{\frac{2}{3}} = x^{\frac{4}{3}} = \sqrt[3]{x^4}$

3. Vereinfache!

a) $\left(\frac{32^{\frac{-3}{2}}}{8^{\frac{-3}{2}}}\right)^{-\frac{1}{3}} = \frac{32^{\frac{1}{2}}}{8^{\frac{1}{2}}} = \left(\frac{32}{8}\right)^{\frac{1}{2}} = 4^{\frac{1}{2}} = 2$

b) $\frac{\left(4^3\cdot 5^{\frac{1}{7}}\right)^{\frac{7}{3}}}{4^{3,5}\cdot 64^{0,5}} = \frac{4^7\cdot 5^{\frac{1}{3}}}{4^{3,5}\cdot 64^{\frac{1}{2}}} = \frac{4^7\cdot 5^{\frac{1}{3}}}{4^{3,5}\cdot 8} = \frac{4^7\cdot 5^{\frac{1}{3}}}{4^{3,5}\cdot 2\cdot 4} = \frac{1}{2}\cdot 4^{2,5}\cdot 5^{\frac{1}{3}} = \frac{1}{2}\cdot 4^{\frac{5}{2}}\cdot 5^{\frac{1}{3}} = \frac{1}{2}\cdot\sqrt{1024}\cdot 5^{\frac{1}{3}} = 16\cdot 5^{\frac{1}{3}}$

c) $\sqrt[6]{\sqrt{66}-\sqrt{2}}\cdot\sqrt[6]{\sqrt{66}+\sqrt{2}} = \sqrt[6]{66-2} = \sqrt[6]{64} = 2$

4. Vereinfache!

$\sqrt[3]{135}+\sqrt[3]{320}+\sqrt[3]{5}$

$=\sqrt[3]{27\cdot 5}+\sqrt[3]{8\cdot 8\cdot 5}+\sqrt[3]{5}$

$=3\cdot\sqrt[3]{5}+4\cdot\sqrt[3]{5}+\sqrt[3]{5}$

$=8\cdot\sqrt[3]{5}$

5. Zeige, dass folgende Aussagen gelten:

a) $\sqrt[n]{x^m}\cdot\sqrt[p]{x^q}=x^{\frac{mp+qn}{np}}$

$$\sqrt[n]{x^m}\cdot\sqrt[p]{x^q}=x^{\frac{m}{n}}\cdot x^{\frac{q}{p}}=x^{\frac{m}{n}+\frac{q}{p}}=x^{\frac{mp}{np}+\frac{qn}{np}}=x^{\frac{mp+qn}{np}}$$

b) $\frac{\sqrt[n]{x^m}}{\sqrt[q]{x^p}}=\sqrt[nq]{x^{mq-np}}$

$$\frac{\sqrt[n]{x^m}}{\sqrt[q]{x^p}}=\frac{x^{\frac{m}{n}}}{x^{\frac{p}{q}}}=x^{\frac{m}{n}-\frac{p}{q}}=x^{\frac{mq}{nq}-\frac{pn}{nq}}=x^{\frac{mq-np}{nq}}=\sqrt[nq]{x^{mq-np}}$$