1. Kürze soweit wie möglich (Versuche immer Zähler und Nenner durch Ausklammern, Faktorisieren oder Umwandlung in binomische Formeln zu vereinfachen!)

a) $\frac{6x^2+12}{3x}=\frac{3\cdot\left(2x^2+4\right)}{3x}=\frac{2x^2+4}{x}=2x+\frac{4}{x}$

b) $\frac{3x}{x^2-x}=\frac{3\cdot x}{x\cdot(x-1)}=\frac{3}{x-1}$

c) $\frac{x^2-2x}{x^2-4}=\frac{x\cdot(x-2)}{(x+2)\cdot(x-2)}=\frac{x}{x+2}$

d) $\frac{36a^2}{24a^4}=\frac{3}{2a^2}$

e) $\frac{63uv}{18(uv)^2}=\frac{7}{2uv}$

f) $\frac{x^2+2xy+y^2}{x^2-y^2}=\frac{(x+y)^2}{(x+y)\cdot(x-y)}=\frac{x+y}{x-y}$

2. Berechne

a) $\frac{3y^2-2}{2y}+\frac{y^2-4}{2y}=\frac{4y^2-6}{2y}=\frac{2\cdot\left(2y^2-3\right)}{2y}=\frac{2y^2-3}{y}=2y-\frac{3}{y}$

b) $\frac{3a-7}{4a^2}-\frac{5a-7}{4a^2}=\frac{-2a}{4a^2}=-\frac{1}{2a}$

c) $\frac{5u+3}{(u-1)^2}-\frac{u+7}{(u-1)^2}=\frac{4u-4}{(u-1)^2}=\frac{4\cdot(u-1)}{(u-1)^2}=\frac{4}{u-1}$

d) $\frac{2}{y}-\frac{1}{2y}=\frac{4}{2y}-\frac{1}{2y}=\frac{3}{2y}$

e) $\frac{3}{x}+\frac{4}{x^2}+\frac{x}{2}=\frac{3x}{x^2}+\frac{4}{x^2}+\frac{x^3}{2x^2}=\frac{6x+8+x^3}{2x^2}=\frac{x^3+6x+8}{2x^2}$

f) $\frac{2}{1-a}-\frac{3}{a-1}=-\frac{2}{a-1}-\frac{3}{a-1}=-\frac{5}{a-1}$

3. einfache Gleichungen

a) 4(r + 2) = 12 | :4

r + 2 = 3 | -2

r = 1 $L = \{1\}$

b) 7,5 = 3(y – 1,5)

7,5 = 3y – 4,5 |+4,5

3y = 12 |:3

y = 4 $L = \{4\}$

c) 11s – 7 = 11s – 3

-7 = -3 |-11s

nicht lösbar ! $L = \{\ \}$

d) 4x – 3 = 2x + 1 |-2x |+3

2x = 4 | :2

x = 2 $L = \{2\}$

e) 7 - 8z = 5 – 2 z |+8z |-5

6z = 2 | :6

z = $\frac{1}{3}$ $L = \left\{\frac{1}{3}\right\}$

f) 3y + 18 = 8y + 8 |-3y |-8

5y = 10 | :5

y = 2 $L = \{2\}$

g) 4(x -1) = 2(x + 1) | :2

2x – 2 = x + 1 |-x |+2

x = 3 $L = \{3\}$

h) 2,5y + 9 –y =4·(1,5 – 0,5y) +17 | zusammenfassen

2,5y + 9 – y = 6 – 2y + 17 |+2y | -9

3,5y = 14 | :3,5

y = 4 $L = \{4\}$

4. Bruchgleichungen – Vergleiche immer das Ergebnis mit der Definitionsmenge!

a) D = R \ {0; 1}

$$\frac{2}{x-1} - \frac{1}{2x} = \frac{1}{6x}$$

$$\frac{2}{x-1} = \frac{1}{6x} + \frac{1}{2x}$$

$$\frac{2}{x-1} = \frac{1}{6x} + \frac{3}{6x}$$

$$\frac{2}{x+1} = \frac{4}{6x}$$

$$\frac{2}{x+1} = \frac{2}{3x} \quad \text{| Kehrwert}$$

$$x+1 = 3x$$

$$1 = 2x$$

$$x = \frac{1}{2} \qquad L = \left\{\frac{1}{2}\right\}$$

b) D = R \ {0; 1}

$$\frac{1}{x-1} = \frac{2}{x}$$

Kehrwert bilden!

$$x-1 = \frac{1}{2}x$$

$$\frac{1}{2}x = 1$$

$$x = 2$$

$$L = \{2\}$$

c) D = R \ -2

$$\frac{5}{x+2} = \frac{3}{2}$$

$$\frac{x+2}{5} = \frac{2}{3} \qquad |\cdot 5$$

$$x+2 = \frac{10}{3} \qquad |-2$$

$$x = \frac{10}{3} - \frac{6}{3}$$

$$x = \frac{4}{3}$$

$$L = \left\{\frac{4}{3}\right\}$$

d) $\frac{1}{x^2+2x}-\frac{1}{(x-1)\cdot(x+2)}=\frac{1}{x^2-x}$

D = R \ {0; 1; -2}

$\frac{1}{x\cdot(x+2)}-\frac{1}{(x-1)\cdot(x+2)}=\frac{1}{x\cdot(x-1)}$

$\frac{(x-1)}{x(x+2)(x-1)}-\frac{x}{x(x-1)(x+2)}-\frac{(x+2)}{x(x-1)(x+2)}=0$

$(x-1)-x-(x+2)=0$

$x-1-x-x-2=0$

$-x-3=0$

$x=-3$

$L=\{-3\}$

e) $1+\frac{18}{x^2-9}=\frac{x}{x+3}$

D = R \ {3; -3}

$1+\frac{18}{(x+3)\cdot(x-3)}=\frac{x}{(x+3)}$

$\frac{(x+3)\cdot(x-3)}{(x+3)\cdot(x-3)}+\frac{18}{(x+3)\cdot(x-3)}=\frac{x\cdot(x-3)}{(x+3)\cdot(x-3)}$

$x^2-9+18=x^2-3x$

$9=-3x$

$x=-3$

$L=\{\ \}$ da $x=-3$ nicht in der Definitionsmenge enthalten ist!